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A NOTE ON FG- INTERIOR

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ABSTRACT

In this paper we have obtained significant properties of Fuzzy g-Interior of a set in fuzzy generalized topological space.

Keywords: *Fuzzy sets, Fuzzy topology, Generalized fuzzy g-Interior.*

I. INTRODUCTION

The concept of generalized topological spaces was introduced and investigated by A. Csaszar. We introduce a new class of Fuzzy g-Interior of a set in fuzzy generalized topological space Also we investigate some of their basic properties and produced many interesting theorems.

II. PRELIMINARIES

Definition 2.1 Let X be a (non-empty) universal crisp set. A **fuzzy topology** on X is a non empty collection τ of fuzzy sets on X satisfies the following conditions

(i) Fuzzy sets **0** and **1** belong to τ
(ii) Any arbitrary union of members of τ is in τ
(iii) A finite intersection of members of τ is in τ

Here **0** and **1** represent the **Zero Fuzzy Set** and the **Whole Fuzzy set** on X, defined as, $0(x) = 0, \forall\ x \in X$ and $1(x) = 1, \forall\ x \in X$. The pair (X, τ) is called **Fuzzy Topological Space** on X, For Convenience, we shall denote the fuzzy topological space simply as X.

Example 2.1 Let $X = \{x_1, x_2, x_3\}$ be the universal crisp set and λ be a fuzzy set defined on X as $\lambda(x_1) = 0.8,\ \lambda(x_2) = 0.5,\ \lambda(x_3) = 0.2$. Then we can see that the collection $\{0, A, 1\}$ satisfies all the three conditions of fuzzy topology on X. Hence $\tau = \{0, \lambda, 1\}$ is a fuzzy topology on X and $\{X, \tau\}$ is a fuzzy topological space.

Proposition 2.1 Let (X, τ) be a fuzzy topological space and λ let be a fuzzy set in X. Then

(i) ϕ, X are fuzzy closed set in X.
(ii) Arbitrary intersection of fuzzy closed sets is a fuzzy closed set.
(iii) Finite union of fuzzy closed sets is a fuzzy closed set.

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proof : (i) let ϕ and X are fuzzy g-closed set it follow that their complement X and ϕ are fuzzy g-closed set in X.

Proof: (ii). Let (X,τ) be a fuzzy topological space and let $\{\lambda_j\}_{j\in J}$ be the collection fuzzy closed sets in X. Where J is any index set then $\{\lambda_j^c\}_{j\in J}$ is a collection of fuzzy open sets in X. This implies $\left({}_{j\in J}\lambda_j^c \right)^c = \left({}_{j\in J}\lambda_j^{\,c} \right)$ is a fuzzy closed set in X.

(iii). Let λ_1, λ_2 be two fuzzy closed sets in X this means $\lambda_1^{\,c}$ and λ_2^c are fuzzy open sets in X. Therefore $\lambda_1^c \quad \lambda_2^c = (\lambda_1 \quad \lambda_2)^c$ is a fuzzy open set in X. Hence $\lambda_1 \quad \lambda_2$ is a closed set in X.

Definition 2.2 Let (X, τ) be a fuzzy topological space and let λ be a fuzzy set in X. Then closure of fuzzy set λ is denoted by $cl(\lambda)$ and is defined to the intersection of all fuzzy closed sets in X containing λ .

Remark 2.2 : We note that $cl(\lambda) = inf\ \{K : \lambda \leq K, K^c \in \tau\}$ Thus closure of a fuzzy set λ is the smallest fuzzy closed set containing.

Proposition 2.2 Let λ be a fuzzy set in a fuzzy topological space (X, τ), then is λ a fuzzy closed if Cl (λ) = λ

Proof:- Suppose that X is a fuzzy closed set in X. Since Cl(λ) is the intersection of all fuzzy closed sets in X containing λ

And $\lambda \geq \lambda$ follows that Cl(λ)$\leq \lambda$. As we know that $\lambda \leq$Cl(λ). Thus, we find that Cl(λ)= λ Thus we find that Cl(λ)= λ Conversely, suppose that Cl(λ)= λ Then by the definition of closure of fuzzy sets it follows that Cl($\lambda \geq \lambda$) is a fuzzy closed set. Thus λ is a fuzzy closed set in X.

Proposition 2.3 Let (X, τ) be a fuzzy topological space and λ_1, λ_2 be a two fuzzy sets in X. Then ;

i $\quad Cl(\phi) = \phi$

ii $\quad Cl(x) = x$

iii $\quad$ If $\lambda_{1,}\lambda_2$ then $cl(\lambda_1) \subseteq cl(\lambda_2)$

iv $\quad cl(\lambda_1 \quad \lambda_2) = cl(\lambda_1) \quad cl(\lambda_2)$

v $\quad cl(\lambda_1 \quad \lambda_2) \subseteq cl(\lambda_1) \quad cl(\lambda_2)$

vi $\quad cl(cl(\lambda_1)) = cl(\lambda_1)$

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Proposition 2.4: Let X be a topological space and $\{\lambda_j\}_{j\in J}$ be a family of subsets of X. Then

(i) $\quad {}_{j\in J}Cl(\lambda_j) \subseteq Cl(\ {}_{j\in J}\lambda_j)$

(ii) $Cl(\ {}_{j\in J}A_j) \subseteq \ {}_{j\in J}Cl(\lambda_j)$

Definition 2.3: Let (X, τ) be a topological space and let λ be a fuzzy set in X. Then interior of fuzzy set λ is denoted by $int(\lambda)$. and is defined to be the union of all fuzzy open sets in X which are contained in λ.

Remark 2.2 : We note that Int. (λ) = Sup {O:O$\leq\lambda$, $O\epsilon\,\tau$). Thus interior of a fuzzy set λ is the largest fuzzy open set contained in λ

Proposition 2.5 Let (X, τ) be a fuzzy topological space and λ_1, λ_2 be two fuzzy sets in X. Then ;

i $\quad Int(\phi) = \phi$

ii $\quad Int(x) = x$

iii $\quad$ If $\lambda_{1,}\lambda_2$ then $Int(\lambda_1) \subseteq Int(\lambda_2)$

iv $\quad Int(\lambda_1 \quad \lambda_2) = int(\lambda_1) \quad Int(\lambda_2)$

v $\quad Int(\lambda_1) \quad int(\lambda_1) \subseteq Int(\lambda_2 \quad \lambda_2)$

vi $\quad Int(Int(\lambda_1)) = Int(\lambda_1)$

proposition 2.6 Let (X,τ) be a fuzzy topological space and let $\{\lambda_j\}_{j\epsilon J}$ be the collection fuzzy closed sets in X. Where J is any index set then

(i) $\quad \cup_{j\epsilon J}\, int\,(\lambda_j) \subseteq int\,(\cup_{j\epsilon J}\lambda_j)$

(ii) $\quad Int(\ \underset{j\in J}{}\lambda_j) \subseteq \ \underset{j\in J}{}Int(\lambda_j)$

Proposition 2.7 Let (X, τ) be a fuzzy topological space and λ let be a fuzzy set in X. Then

(i) $\quad Int\ (1-\lambda) = 1 - Cl(\lambda)$

(ii) $\quad Cl(1-\lambda) = 1 - Int(\lambda)$

Proof:- We have $Int(\lambda) = U_j\lambda_j$ where λ is a fuzzy open set in X and $\lambda_j \leq \lambda, \forall \lambda j \in J$. This implies that $1 - int(\lambda) = 1 - U_j\lambda_j = \ {}_j\lambda^c$, where $\{\lambda^c\}$ is the family of fuzzy closed sets containing $(1-\lambda)$ Further, we have This implies $Int\ (1-\lambda) = 1 - Cl(\lambda)$ Hence, by the definition of closure of fuzzy set we get $Cl(1-\lambda) = 1 - Int(\lambda)$.

<u>**G-interior in generalized fuzzy topological spaces**</u>

Definition 3.1: Let X be a (non-empty) universal set. A fuzzy topology on X is a non empty collection τ_{Fg} of fuzzy sets on X satisfying the conditions

(i) Fuzzy sets **0** and **1** belong to τ

(ii) if $\{\lambda_j\}$ for $j \in J$ is any family of fuzzy sets on X and $\lambda_j \in \tau_{Fg}, \forall j \in J$ then $_{j \in J} \lambda_j \in \tau_{Fg}$

the pair (x, τ_{Fg})is called fuzzy generalized topological . the element of family τ_{Fg} are called fuzzy g-open sets and their complements are called fuzzy g-closed sets.

Definitions 3.2

Let (X, τ_{Fg}) be a topological space and let λ be a generalized fuzzy set in X. Then the g- interior of fuzzy set λ is denoted by . $I_{Fg}(\lambda)$ and is defined to be the union of all fuzzy g-open sets in X which are contained in λ .

Remark 3.2 We note that $I_{Fg}(\lambda)$ = Sup {0:0$\leq \lambda$, $0 \epsilon \tau$). Thus g-interior of a fuzzy set λ is the largest fuzzy g-open set contained in λ

Proposition 3.1 Let λ be a fuzzy set in fuzzy generalized topological space (x, τ_{Fg}) then λ is fuzzy g - open if and only if $I_{Fg}(\lambda) = \lambda$

Proof: Suppose λ is a fuzzy g-open set in X. Since $I_{Fg}(\lambda)$ is the union of all fuzzy g-open sets in X contained in λ and it $\lambda \leq \lambda$ follows that $I_{Fg}(\lambda) \leq \lambda$ As we know that $I_{Fg}(\lambda) \leq \lambda$. Thus we find that $I_{Fg}(\lambda) = \lambda$ Conversely, suppose that $I_{Fg}(\lambda)$ then by definition of g -interior of fuzzy set, it follows that $I_{Fg}(\lambda)$ is a fuzzy g- open set. Thus λ is a fuzzy g-open set in X.

Proposition 3.2 Let (X, τ_{Fg}) be a generalized fuzzy topological space and λ_1, λ_2 be two fuzzy sets in X. Then ;

i $I_{Fg}(\phi) = \phi$

ii $I_{Fg}(x) = x$

iii If λ_1, λ_2 then $I_{Fg}(\lambda) \subseteq I_{Fg}(\lambda)$

iv $I_{Fg}(\lambda_1 \quad \lambda_2) = I_{Fg}(\lambda_1) \quad I_{Fg}(\lambda_2)$

v $I_{Fg}(\lambda_1) \quad I_{Fg}(\lambda_1) \subseteq I_{Fg}(\lambda_2 \quad \lambda_2)$

vi $I_{Fg}(I_{Fg}(\lambda_1)) = I_{Fg}(\lambda_1)$

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proof : (i) since ϕ and X are fuzzy g-open sets from let (x, τ_{Fg}) be a generalized fuzzy topological space and let λ be a fuzzy set in X. then λ is fuzzy g-open set if and only if $I_{Fg}(\lambda) = \lambda$. we have $I_{Fg}(\phi)=\phi$ and $I_{Fg}(X) = X$.

(ii) Suppose $\lambda_1 \subseteq \lambda_2$ in x since $\lambda_2 \subseteq I_{Fg}(\lambda_2)$ and $\lambda_1 \subseteq \lambda_2$ we have $\lambda_1 \subseteq I_{Fg}(\lambda_2)$, now $I_{Fg}(\lambda_2)$ is a fuzzy g-open set and is $I_{Fg}(\lambda_1)$ the largest fuzzy g-open set containing λ we find that

$$I_{Fg}(\lambda_1) \subseteq I_{Fg}(\lambda_2)$$

(iii) Since $\lambda_1 \subseteq \lambda_1 \quad \lambda_2$, $\lambda_2 \subseteq \lambda_1 \quad \lambda_2$ we have $I_{Fg}(\lambda_1) \subseteq I_{Fg}(\lambda_1 \quad \lambda_2)$ and $I_{Fg}(\lambda_2) \subseteq I_{Fg}(\lambda_1 \quad \lambda_2)$ this implies $I_{Fg}(\lambda_1) \quad I_{Fg}(\lambda_2) \subseteq I_{Fg}(\lambda_1 \quad \lambda_2)$

(iv) Since $\lambda_1 \quad \lambda_2 \subseteq \lambda_1$ and $\lambda_1 \quad \lambda_2 \subseteq \lambda_2$ we have $I_{Fg}(\lambda_1 \quad \lambda_2) \subseteq I_{Fg}(\lambda_1) \quad I_{Fg}(\lambda_2)$

(v) since $I_{Fg}(\lambda_1)$ is a g- fuzzy open set in X. if follow that

$$I_{Fg}(I_{Fg}(\lambda_1)) = I_{Fg}(\lambda_1)$$

Example 3.1 let X = {x1, x2} let $\lambda_1, \lambda_2, \lambda_3 \text{and} \lambda_4 \text{be}$ fuzzy sets for X defined as

$\lambda_1 = \{(x_1, 0.4), (x_2, 0.6)$ $\quad \lambda_1^c = \{(x_1, 0.6), (x_2, 0.4)\}$

$\lambda_2 = \{(x_1, 0.5), (x_2, 0.3)\}$ $\quad \lambda_2^c = \{(x_1, 0.5), (x_2, 0.7)\}$

$\lambda_3 = \{(x_1, 0.3), (x_2, 0.4)\}$ $\quad \lambda_3^c, = \{(x_1, 0.6), (x_2, 0.7)$

$\lambda_4 = \{(x_1, 0.5), (x_2, 0.6)\}$ $\quad \lambda_4^c = \{(x_1, 0.5), (x_2, 0.4)\}$

Then $\tau_{Fg} = \{0, \lambda_1, \lambda_2, \lambda_3, \lambda_4, 1\}$ is a fuzzy topology on X. It is easy to see that τ_g^c the collection of closed sets in X is given by

$$I_{Fg}(\lambda_1^c) = (x_1, 0.5), (x_2, 0.3) = \lambda_2$$

$$I_{Fg}(\lambda_2^c) = (x_1, 0.5), (x_2, 0.6) = \lambda_3$$

$$I_{Fg}(\lambda_1) = \{(x_1, 0.4), (x_2, 0.6)\} = \lambda_1$$

and $I(\lambda_2) = \{(x_1, 0.5), (x_2, 0.3)\} = \lambda_2$

$$I_{Fg}(\lambda_1 \quad \lambda_2) = \lambda_3$$

$$I_{Fg}(\lambda_1) \quad I_{Fg}(\lambda_2) = \lambda_4$$
$$I_{Fg}(\lambda_1) \quad I_{Fg}(\lambda_2) = \lambda_4$$
$$I_{Fg}(\lambda_1) \quad I_{Fg}(\lambda_2) \neq I_{Fg}(\lambda_1 \quad \lambda_2)$$

proposition 3.3 Let (X, τ_{Fg}) be a generalized fuzzy topological space and let $\{\lambda_j\}_{j\in J}$ be the collection fuzzy g-open sets in X. Where J is any index set then

(i) $_{j\in J} I_{Fg}(\lambda_j) \subseteq I_{Fg}(\ _{j\in J}\lambda)$

(ii) $I_{Fg}(\ _{j\in J}(\lambda_j) \subseteq \ _{j\in J} I_{Fg}(\lambda)$

Proposition 3.4 Let (X, τ_{Fg}) be a generalized fuzzy topological space and λ let be a fuzzy set in X. Then

(i) $I_{Fg}\ (1-\lambda) = 1 - C_{Fg}(\lambda)$

(ii) $C_{Fg}(1-\lambda) = 1 - I_{Fg}(\lambda)$

Proof:- We have $C_{Fg}(1-\lambda) = 1 - I_{Fg}(\lambda)$ where λ is a fuzzy open set in X and $\lambda_j \leq \lambda, \forall \lambda j \in J.$ This implies that $1 - I_{Fg}(\lambda) = 1 - U_j \lambda_j = \ _j\lambda^c$, where $\{\lambda^c\}$ is the family of fuzzy g-open sets containing $(1-\lambda)$ Further, we have This implies $I_{Fg}(1-\lambda) = C_{Fg}(1-\lambda)$. Hence, by the definition of g-open of fuzzy set we get $C_{Fg}(1-\lambda) = 1 - I_{Fg}(\lambda)$.

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